ABSTRACTS

THERMAL CONDUCTIVITY OF BENZENE AND TOLUOL AT DIFFERENT TEMPERATURES AND PRESSURES

N. F. Potienko and V. A. Tsymarnyi UDC 536.222

We determined the experimental coefficients of thermal conductivity of benzene and toluol at temperatures of up to 150 and 200°C, respectively, and pressures of up to 49.0 MN/m^2. The measurements were made to check the reliability of the apparatus described previously [1], using the nonsteady-state method with a linear heat source of constant power to obtain new experimental data at elevated temperatures and pressures.

Expansion of the working-parameter range required a change in a number of the units described earlier [1]. The cell was heated with a copper block bearing a Nichrome heating element and a photothyratron regulator. The temperature in the block was measured with a platinum resistance thermometer (PTS-10) and an R-308 potentiometer. Control measurements showed that the change in temperature in the cell did not exceed 10^{-3} deg/min. Since the duration of the experiment was about 10 sec and the maximum rise in heat-source temperature did not exceed one degree, this temperature-regime stability was quite adequate. The temperature gradient over the height of the cell was no greater than 10^{-4} deg/mm.

The measuring-cell design made it possible to conduct investigations at pressures of up to 49.0 MN/m^2.

At elevated temperatures, the source power could be reduced to 0.3 W/m and the sensitivity of the recording system increased to 0.01 deg/mm. In this case, the value of the product Gr · Pr [1], modified for nonsteady-state processes, did not exceed 1000.

The experimental values of λ for benzene and toluol were characterized by an average divergence from smooth curves of ±0.5%; the maximum deviation was 1.8% and the calculated error of the method was 2.5%.

The values of λ obtained for toluol and benzene at their saturation pressures were compared with the results of other researchers. In most cases, the discrepancies were ±2.5-3.0%. In other studies [3, 4], the deviation at the limits of the temperature range investigated reached 4%. The data of Rastorguev et al. [2] are 4-8% lower than ours at atmospheric pressure and 10-12% lower at higher pressures. The discrepancy for benzene was 2-3% in most cases.

Tables not included in this summary give the smoothed values of λ for benzene and toluol obtained by graphic processing of the experimental data. The coefficients of thermal conductivity λ for benzene and toluol at atmospheric pressure and 30°C were 0.1426 and 0.1318 W/m · deg, respectively.

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EQUATION OF STATE AND THERMODYNAMIC PROPERTIES OF CARBON DIOXIDE UP TO A PRESSURE OF 2500 BARS

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UDC 536.71

The article presents new experimental data on the density of liquid carbon dioxide obtained with a device that has been described in [1]. On the basis of these quantities and the values of density taken from [2], we construct a reference grid, which is described analytically with high accuracy by an equation of state having the form

$$P = A(T)\rho + B(T)\rho^3 + C(T)\rho^5 + D(T)\rho^7. \qquad (1)$$

The temperature functions in Eq. (1) are found by the method of squaring of isotherms using operations similar to those described in [3], and are approximated by the equations

$$\begin{aligned} A(T) &= 2387.3 + 3.1525T - 16169\cdot 10^2 T^{-1} + 20186\cdot 10^4 T^{-2}, \\ B(T) &= 2316 - 10.02T + 4032\cdot 10^2 T^{-1} - 8885\cdot 10^4 T^{-2}, \\ C(T) &= -4640 + 15.6T, \quad D(T) = 1860 - 4.5T. \end{aligned} \qquad (2)$$

In the equation of state, the pressure is in bars, and the density is, kg/dm^3.

A comparison of the calculated density values and the reference values showed that the standard deviations on the isotherms are 0.02-0.06%, and the maximum values are 0.05-0.13%. The deviation from the experimental values of density, as a rule, is within the limits ±0.05%, and it reaches 0.10-0.13% only for five points. The equation quite satisfactorily represents the data on the density of liquid on the curves of saturation and solidification from the temperature of the triple point to 10 and −40°C, respectively.

Based on the equation of state, we calculate tables (given in the article) on the thermodynamic properties of carbon dioxide in the temperature range 220-320°K up to a pressure of 2500 bars. In order to calculate the calorific values, we integrate over the limits from the density of the boiling liquid to densities corresponding to the given pressures. Deviations of the calculated values of the isobaric specific heat from the experimental values [4] do not exceed 2%, decreasing in proportion to distance from the critical region. We also observe satisfactory agreement with the data of [5] based on the adiabatic differential Joule–Thomson effect.

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Odessa Institute of Maritime Engineers. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 20, No. 4, p. 734, April, 1971. Original article submitted May 4, 1970; abstract submitted July 20, 1970.

APPARATUS OPERATING IN STEADY STATE FOR THERMOPHYSICAL STUDIES IN GASEOUS MEDIUM AT HIGH PRESSURES AND TEMPERATURES

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UDC 536.62

In this article are presented a diagram and the construction of apparatus designed for conducting a wide range of physicochemical studies on gaseous media at pressures up to 12 kbar and temperatures to 3000°K.

A diagram of the apparatus, which allows unrestricted retention for a long time of a fixed temperature in an inert gas compressed at the corresponding pressures, is presented in Fig. 1.

The apparatus consists of a thick-walled force cylinder (1) calculated for the necessary inner pressure. The inner wall of the force cylinder is separated from the thermal chamber (2) by a thermal insulation layer (3). A water cooling system of channels (4) is distributed in the thick wall of the force cylinder. Heating of the inner volume of the thermal chamber is accomplished with an electric heating element (5). "Thermal locks" (6) were set in the ends of the chamber so that the heat conduction along the length of the apparatus would not have a significant effect on the equilibrium nature of the temperature distribution along the axis of the thermal chamber.

The results of thermal and endurance calculations of numerous variants of the apparatus operating according to the diagram presented showed that the most expedient of these conditions proved to be the use of pyrolytic graphite as the thermal insulating material, which has both a comparatively low thermal conductivity (0.7-1.1 W/m · deg), and an insignificantly small thermal expansion coefficient.

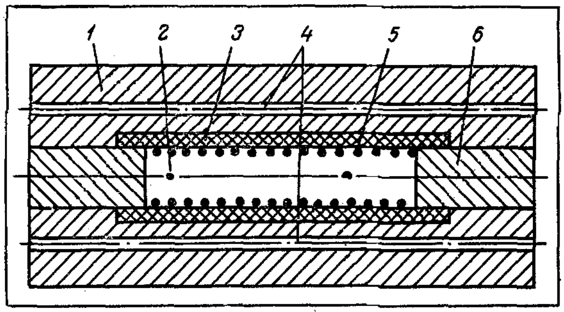

Fig. 1. Diagram of apparatus

The inert or neutral gas studied (argon, nitrogen, etc.) is pumped into the chamber under a preliminary pressure of 1-5 kbar using a gas compressor of Academician L. F. Vereshchagin's construction. Subsequent increase in the gas pressure is accomplished by heating.

Multiple tests were conducted on an experimental model of the apparatus on nitrogen at pressures up to 6 kbar and temperatures above 2000°K. The results of the tests indicate the reliability of the design and the full efficiency of all the construction elements of the apparatus.

Institute of High-Pressure Physics, Academy of Sciences of the USSR, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 20, No. 4, pp. 734-735, April, 1971. Original article submitted October 20, 1969; abstract submitted August 18, 1970.

CALCULATION OF THE VISCOSITY OF REAL GASES ON THE BASIS OF THE LAW OF CORRESPONDING STATES

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UDC 533.16

A method for calculating the viscosity coefficient of insufficiently investigated substances on the basis of the law of corresponding states is proposed. The characteristic volume and temperature values, which are generally variable, are used as the reduction parameters rather than the critical parameters ordinarily used in the well-known methods. These characteristic values are determined by using experimental data on the viscosity of the rarefied gas of the substance under investigation and of another, thoroughly investigated, substance, which is used as a standard.

The method can be used for generalizing with sufficient accuracy the experimental data on the viscosity of rarefied gases with any molecular structure – from the simple "spherical" to complex polar gases. Moreover, it is shown that these data can be reliably extrapolated to the temperature range determined by the availability of data on the viscosity of the standard.

The obtained reduction parameters are used to form reduced temperature and density as independent variables if it is necessary to calculate the viscosity coefficient of compressed gases. The calculation is then performed by means of the equation written for the standard substance.

The above method was used for calculating the viscosity coefficient of heavy-water vapor in the temperature range 300-550°C at pressures up to 500 bar. The mean error of our results amounts to 1-3%, while the maximum error does not exceed 6%.

DISPERSION OF DROPS OF CONDUCTING LIQUIDS IN A DIELECTRIC MEDIUM DURING RECHARGING ON ELECTRODES IN AN ELECTRIC FIELD

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UDC 532.529.6

The article, which is a continuation of [1], presents analytical evaluations of the values of charges and conditions of dispersion of drops of conducting liquids in a dielectric medium in a uniform electric field during recharging on electrodes, and also the results of experiments on dispersion on recharging drops of electrolytes and surface-active agents (Table 1) in mineral oil.

The formula for the charge q of a drop, which is approximated by an ellipsoid of revolution similar to that which was done in [2] on the assumption that the surface of a semispheroid, which simulates the drop, and its semimajor axis are equal to the surface and semimajor axis of the corresponding ellipsoid, has the form

$$|q| \simeq \frac{2\pi\varepsilon\varepsilon_0 r^2 E_0\left(\sqrt{1+(1-e^2)+(1-e^2)^2}-1\right)}{\sqrt[3]{(1-e^2)^2}\,(\eta_e^2-1)\,(\eta_e \operatorname{arcth}\eta_e-1)}, \tag{1}$$

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TABLE 1. Values of α and k for Drops of Investigated Solutions

$\alpha \cdot 10^3$, N/m	3,6	4,0	12,0	32,2	54,0	57,5
k	0,40	0,40	0,34	0,35	0,32	0,31

where $\eta_e \simeq \sqrt{2}/\sqrt{3-\sqrt{1+(1-e^2)+(1-e^2)^2}}$, $e=\sqrt{1-b^2/a^2}$; r is the radius of the drop; a, b, e are the semiaxes and eccentricity of the ellipsoid; α is the coefficient of interphase surface tension; E_0 is the external field strength; $\varepsilon\varepsilon_0$ is the dielectric constant of the medium. The correspondence of this formula to the experimental data and to the calculation [3] for the case $e \to 0$ is indicated.

Substitution of the value of q obtained into Eq. (6) from [1] permits obtaining the condition of dispersion of recharging drops in the form

$$E_0 \geqslant E_0^{cr} = k\,(\alpha/\varepsilon\varepsilon_0 r)^{1/2}, \tag{2}$$

where E_0^{cr} is the critical value of E_0 corresponding to dispersion of the drops, $k = f(e)$. It is assumed that $e = 0.952$ for critical conditions [1]. Then $k = f(e)$ is determined numerically: $k = 0.3$.

A description of the experiments is given, from which follows that the values of k (Table 1) found from the average experimental data with a mean error of not more than 12% for drops of all investigated solutions coincide, so that the arithmetic mean value $k_{av} = 0.35 \pm 0.03$. This result is regarded as satisfactory, and Eq. (2), where $k = 0.3$, is recommended for practical use.

It follows from Eq. (2) that the magnitude of the charge and intensity of dispersion of the recharging drops, other conditions being equal, do not depend on the chemical composition.

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A METHOD OF DETERMINING THE ELASTIC AND INELASTIC PROPERTIES OF SMALL SPECIMENS OF MATERIALS AT HIGH TEMPERATURES

V. M. Baranov and Yu. V. Miloserdin

UDC 620.17

An ultrasonic resonance-pulse method for determining the elastic constants and the oscillation energy scattering characteristics of small specimens of isotropic materials is described. Measurements are made on specimens in the form of disks of diameter from 6-8 mm to 50-60 mm and thickness from 1 to 15 mm, for a thickness to diameter ratio from 0.1 to 0.25. In this method several of the lower natural frequencies of oscillation of the specimens are measured, from which the normal modulus of elasticity the shear modulus, and Poisson's ratio are calculated. From the damping decrement of the oscillations the viscosity of the materials can be found.

The apparatus for making these measurements over a wide temperature range consists of two sound-conducting rods of diameter 6 mm and length 600 mm, between which the specimen is clamped. A piezoelectric radiator and a piezoelectric receiver are fixed to the free ends of the rods. The piezoelectric

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radiator is excited 10-30 times/sec by high-frequency pulses of duration 200-300 μsec, with a variable filling frequency. The signals from the piezoelectric receiver are amplified and observed on the screen of an oscilloscope. The effect of clamping the specimen on the intensity of excitation of the natural frequencies corresponding to different shapes of the oscillations and on the measurement accuracy is considered. The method of calculating the elastic constants from the results of the measurements is described in detail, and appropriate theoretical tables are presented. The accuracy of determining the moduli of elasticity is estimated to be within 2-2.5%.

As an illustration of the use of this method results are presented for the elastic constants and viscosity of specimens of heat-resistant constructional steels Kh18N9T, ÉI-787, and VZh-98, in the temperature range 293-1200°K, made on specimens of diameter 18 mm and thickness 3 mm in the frequency range 130-210 kHz. From the values of the elastic moduli obtained the velocities of propagation of longitudinal and transverse elastic waves are calculated. The activation energy of the processes connected with the absorption of oscillation energy, calculated from the viscosity–temperature curve, is 1.1, 1.15, and 1.4 eV, respectively, for the above materials.

FLOW OUTSIDE THE TURBULENT REGION OF AN AXISYMMETRIC JET DISCHARGING INTO A SEMIBOUNDED SPACE

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UDC 532.522

Flow outside the region of an axisymmetric jet was considered earlier by L. D. Landau on the assumption of potentiality of motion. It can be shown, however, that the character of a secondary flow induced by an axisymmetric turbulent jet can be investigated without the assumption of potentiality. The smallness of the velocities of the secondary flow permit neglecting the inertia terms in the Navier–Stokes equations and reduces the problem to integration of approximate Stokes equations:

$$\frac{1}{\rho}\cdot\frac{\partial p}{\partial P}=\frac{\nu}{R^2\sin\theta}\cdot\frac{\partial D\psi}{\partial\theta};\qquad \frac{1}{\rho}\cdot\frac{\partial p}{\partial\theta}=-\frac{\nu}{\sin\theta}\cdot\frac{\partial D\psi}{\partial R}.$$

The solution is sought in the form: $\psi = Rf(\cos\theta)$. The constants of integration are determined from the conditions on the boundaries of the region: the conditions when $\theta = \theta_1$ reflect adhesion of the liquid on the nozzle wall, i.e., vanishing of both velocity components; the conditions when $\theta = \theta_0$ reflect closing of the solutions in the turbulent jet and in the investigated region.

The solution of the problem of an axisymmetric turbulent source with a "new" Prandtl dependence for turbulent shear stress is used for flow in a turbulent jet.

On the basis of the analytical solution obtained for the stream function the calculated streamlines and distribution of the peripheral velocity of the secondary flow in the region ($\theta_1 = 45°$, $\theta_0 = 12°$) are constructed.

Rarefaction of the pressure, as the solution shows, decreases with distance from the source (as $1/R^3$), whereas with a potential character of the secondary flow the rate of decrease is less (as $1/R^2$).

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CONVECTIVE HEAT TRANSFER IN A PRISMATIC PIPE OF TRIANGULAR SECTION

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Heat transfer during laminar flow of a viscous incompressible liquid inside a straight semiinfinite prismatic pipe (channel) with a cross section in the form of an equilateral triangle with consideration of friction and other internal sources of heat was investigated.

The problem is reduced to the solution of the equation

$$W(x, y)\frac{\partial T}{\partial z} = a\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\dot{q}(x, y, z)}{c\gamma} + \frac{\mu}{c\gamma}\,\text{Diss. Fkt.}\left(\frac{\partial W}{\partial x}, \frac{\partial W}{\partial y}\right) \tag{1}$$

with boundary conditions

$$T(x, y, z)|_{z=0} = T_0 = \text{const}, \quad T(x, y, z)|_\Gamma = f(z), \tag{2}$$

where Γ is the boundary of the triangular region.

In dimensionless coordinates the equation and boundary conditions have the form

$$W(\xi, \eta)\frac{\partial T}{\partial \zeta} = \left(\frac{\partial^2 T}{\partial \xi^2} + \frac{\partial^2 T}{\partial \eta^2}\right) + \frac{b^2 \dot{q}(\xi, \eta, \zeta)}{\lambda} + \frac{\mu}{\lambda}\left[\left(\frac{\partial W}{\partial \xi}\right)^2 + \left(\frac{\partial W}{\partial \eta}\right)^2\right], \tag{3}$$

$$T(\xi, \eta, \zeta)|_{\zeta=0} = T_0, \quad T(\xi, \eta, \zeta)|_\Gamma = f(\zeta), \tag{4}$$

where b is the altitude of the equilateral triangle.

The Laplace integral transform with respect to the longitudinal coordinate ζ is used. The problem obtained in the region of the transforms is solved by the Bubnov–Galerkin method.

The final solution is presented in the form of the sum of products of polynomials and exponential functions, whereby the polynomials depend on the variables ξ and η, and the exponential functions only on ζ. The following cases are investigated in detail.

1. A constant temperature regime is maintained on the pipe wall and the liberation of heat by internal sources is uniform over the entire volume of the flow. The solutions are found in the second and third approximations. Simple and sufficiently accurate formulas are obtained for determining local Nusselt numbers (in the absence of internal sources and energy dissipation). The change of the Nusselt number along the flow is presented graphically.
2. The temperature at the pipe entrance coincides with the wall temperature and the power of the internal heat sources is negligibly small, i.e., heat transfer is due only to the heat of friction. Stabilization of the temperature profile along the liquid flow (on the plane of the meridian section) is presented graphically. Formulas are obtained for the relative heat flux on the pipe walls. Investigations of the formulas obtained showed that the heat flux in the middle of the face is maximum and in the corner regions is minimum, which is consistent with the data of other authors.
3. The wall temperature is a linear function of the longitudinal coordinate and the power of the internal heat sources is constant over the entire volume of the flow. A graph of temperature stabilization along the liquid flow in the absence of internal sources and energy dissipation is presented.
4. The wall temperature increases exponentially, and the power of the internal heat sources is constant.

The convergence of the approximate solutions to the exact solution is discussed.

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A METHOD OF SOLVING HEAT CONDUCTION PROBLEMS INVOLVING NONLINEAR HEAT-TRANSFER RELATIONS ON THE BOUNDARIES OF A BODY

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UDC 536.2.01

We consider the solution of the heat conduction differential equation

$$R^2 \frac{\partial T}{\partial \mathrm{Fo}} = \frac{\partial^2 T}{\partial x^2} + \frac{k-1}{x} \cdot \frac{\partial T}{\partial x}, \qquad 0 \leqslant x \leqslant R, \tag{1}$$

for the boundary conditions

$$T(x, 0) = T_0, \quad \frac{\partial T}{\partial x}(0, \mathrm{Fo}) = 0, \tag{2}$$

$$\lambda \frac{\partial T}{\partial x}(R, \mathrm{Fo}) = q\,[T(R, \mathrm{Fo})]. \tag{3}$$

For the calculation of temperature fields of bodies, for example, the temperature fields in industrial furnaces, the boundary condition (3) is nonlinear:

$$\lambda \frac{\partial T}{\partial x}(R, \mathrm{Fo}) = \alpha\,[T_b - T(R, \mathrm{Fo})] + \sigma\,[T_r^4 - T^4(R, \mathrm{Fo})]. \tag{4}$$

An exact solution of the system (1), (2), and (4) is not known.

In this connection, it may be possible to have an approximate solution, which satisfies exactly only the heat conduction equation and the initial conditions. In such a solution the boundary conditions are satisfied discretely at g time instants, taken on the interval Fo, the evolution time of the process. The quantity g determines the degree of approximation to the exact solution, and as $g \to \infty$, the approximate solution becomes coincident with the exact solution.

An approximate solution can be obtained, which is based on a generalized notation for the temperature field in bodies for boundary conditions of the first kind. We approximate the temperature variation law on the surface of the body by a polynomial

$$T(R, \mathrm{Fo}) = T_0 + \sum_{n=1}^{n=g} A_n \mathrm{Fo}^{\frac{n}{2}}. \tag{5}$$

For a plate the problem reduces to solving a system of g equations in g unknowns A_n:

$$2 \frac{\lambda}{R} \sum_{n=1}^{n=g} A_n \Phi_{1,n}(\mathrm{Fo}_i) = q(\mathrm{Fo}_i), \quad i = 1, 2, 3, \ldots, g. \tag{6}$$

To solve the system we obtain the constants A_n. The temperature on the plate surface is determined from Eq. (5); the temperature at the center, and the temperature averaged with respect to the mass, are given, respectively, by the equations

$$T(R, \mathrm{Fo}) - T(0, \mathrm{Fo}) = \sum_{n=1}^{n=g} A_n \Phi_{4,n}(\mathrm{Fo}), \tag{7}$$

$$T_{av}(\mathrm{Fo}) - T_0 = 2 \sum_{n=1}^{n=g} A_n \Phi_{2,n}(\mathrm{Fo}). \tag{8}$$

Nomograms are then constructed for the functions $\Phi_{1,n}(\mathrm{Fo})$, $\Phi_{2,n}(\mathrm{Fo})$, $\Phi_{4,n}(\mathrm{Fo})$.

Studies show that it is sufficient to restrict g to the value g = 2 for thermally thin bodies and g = 3 for massive bodies. Moreover the error of the method ≈1.5%. The method can be used for compound and nonlinear boundary conditions, and also for boundary conditions in implicit form.

V. I. Lenin Ivanovskii Energetics Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 20, No. 4, pp. 739-740, April, 1971. Original article submitted September 9, 1969; abstract submitted July 1, 1970.

ACCURACY OF THE SOLUTION OF BOUNDARY-VALUE PROBLEMS ON COMBINED MODELS

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UDC 681.142.334

The solution of two- and three-dimensional boundary-value problems can be obtained by the method of electrical modeling with the use of combined models in which the physical field is represented by a continuous electrically conducting medium in combination with discrete elements. This class includes in particular models of the type "electrically conducting paper-resistors" used for solving problems of unsteady heat conduction by the method of successive intervals (Liebmann method).

The connection of resistors with conducting paper at nodes is accomplished via contact electrodes of finite size which are generally round with a diameter $\overline{D} = 0.1$ of the spacing of the nodes. An increase of the current density in the neighborhood of the electrodes is accompanied by distortion of the potential lines in the continuous medium, which introduces a certain error into the solution.

To reduce the error of the discrete current supply, it is suggested to use models with relatively enlarged dimensions of the contact electrodes $\overline{D} = 0.3$ with average values of the space and time intervals.

The change of the effective resistance of the conducting paper in the presence of the enlarged contact electrodes can be taken into account by introducing a correction into the value of the specific resistance of the conducting paper, which enters into the formulas for calculating the discrete elements of the model, particularly the time resistors R_τ.

The magnitude of the correction can be determined from the experimental relations between the dimensionless effective resistance of a square of conducting paper and the size and shape of the contact electrodes.

Another way of reducing the effect of discreteness is to change the magnitude of the resistors supplied to the inside nodes. The magnitude of the correction for the effect of discreteness can be determined experimentally. However, in this case it is recommended to use enlarged electrodes in the boundary regions.

It is shown with reference to two-dimensional problems for which the theoretical solution is known that the methods proposed in the article for reducing the effect of discreteness of electrodes permit reducing the modeling error from 8-7 to about 2% of the maximum potential difference for average values of the space and time intervals. Modeling was done on the ÉGDA-9/60 electrointegrator with a specialized attachment for solving two-dimensional problems of unsteady heat conduction.

V. I. Lenin Ivanovo Power Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 20, No. 4, pp. 740-741, April, 1971. Original article submitted January 29, 1970; abstract submitted August 3, 1970.

PRINCIPAL BOUNDARY-VALUE PROBLEMS OF THERMOELASTICITY FOR A CIRCULAR DISK

D. V. Grilitskii and I. N. Osiv

UDC 539.377

The authors investigated a circular isotropic disk referred to a rectangular Cartesian coordinate system with the origin at the center of the disk, at some point of which is located a stationary concentrated heat source with intensity W.

Mixed conditions for the temperature $T(z, \bar{z})$ are assigned at the edge of the disk

$$\frac{\partial T(t, \bar{t})}{\partial n} = f_1(t), \quad t \in L_1, \tag{1}$$

$$T(t, \bar{t}) = f_2(t), \quad t \in L_2, \tag{2}$$

and homogeneous conditions for stresses

$$\sigma_r + i\tau_{r\theta} = f(t), \qquad t \in L, \tag{3}$$

or for displacements

$$u + iv = g(t), \qquad t \in L. \tag{4}$$

The bases of the disk are assumed heat-insulated.

The stationary temperature field and stresses of the disk are determined by means of the relationships of the two-dimensional theory of thermoelasticity [1, 2].

The function determining the temperature in the disk is presented in the form

$$T(z, \bar{z}) = \mathrm{Re}\,[A \ln (z - \bar{z}_0) + F_0(z)]. \tag{5}$$

Determination of the function of the complex variable $F_0(z)$ holomorphic in the region of the disk is reduced to a singular integral equation of the first kind whose solution in the general case is represented in quadratures.

The thermoelastic state for a half-plane under the effect of a concentrated stationary heat source, both with boundary conditions of the mixed type for temperature and homogeneous for the mechanical characteristic was obtained by passage to the limit. For certain particular values of the initial data the results of the article coincide with those obtained earlier in [3, 4].

The thermoelastic state of the disk and half-plane in real variables is determined, and graphs of the distribution of temperature and stresses at the edge of the disk are also presented.

NOTATION

$z = x + iy$	is the complex variable;
$f(t)$, $f_1(t)$, $f_2(t)$, $g'(t)$	are the known functions satisfying condition H;
$\partial/\partial n$	is the derivative with respect to the normal;
$t = x + iy$	is a point located at the edge of the disk;
$L_1 = \sum_{j=1}^{n} a_j b_j$	is the set of arcs on L arranged so that, by-passing L counterclockwise, points a_1, b_1, a_2, b_2, . . . , a_n, b_n are found in the indicated sequence;
$L_2 = L - L_1$;	
σ_r, $\tau_{r\theta}$	are the normal and tangential stress components on L;
u, v	are the radial and tangential displacements of points of the edge L;
A	is a coefficient determined by the intensity of the heat source, thickness, and coefficient of internal heat conductivity of the disk.

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